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Bifurcation analysis of steady Rayleigh–Bénard convection in a cubical cavity with conducting sidewalls

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Natural convection in a cubical cavity heated from below with perfectly conducting sidewalls is investigated numerically. A parameter continuation procedure based on a Galerkin spectral method was applied to obtain the bifurcation diagrams for steady flow solutions over the region of Rayleigh numbers $Ra \leqslant 1.5 \times 10^5$ at Prandtl numbers $Pr = 0.71$ and 130. In both cases, the bifurcation diagrams were more complex than those previously reported for adiabatic sidewalls. Four and nine different convective solutions (without taking into account the solutions obtained by symmetry) that were stable over certain ranges of Ra were respectively identified at $Pr = 0.71$ and 130. The dependence of the bifurcation diagrams and of the topology of the flow patterns on the Prandtl number were also stronger in the case of conducting sidewalls. Most of the flow patterns investigated evolved to double toroid-like topologies with increasing Rayleigh number. This is especially noticeable at $Pr = 130$, where all flow patterns adopted double-toroid shapes that were superimposed on the characteristic flow patterns observed at values of Ra slightly above the respective bifurcation points where they originated. At sufficiently high Ra the double-toroid pattern configuration prevailed. This phenomenon, which has not been previously observed in the case of adiabatic lateral walls, is related to the thermal activity of the sidewalls, which locally extract/supply relatively large amounts of heat from/to the fluid. These predictions are consistent with experimental flow transitions and topologies reported in the literature. In addition, a complete bifurcation study in the two-dimensional (Ra, Pr)-plane was carried out for the flow pattern with an initial configuration of four connected half-rolls which was stable at both $Pr = 0.71$ and 130. Since the surface of Nu over the (Ra, Pr)-plane presented several folds and cusps, different regions were identified as a function of the number of particular realizations of this flow pattern, varying between zero and five. Three different regions of stability were identified for this particular flow pattern in the (Ra, Pr)-plane within the range of parameters investigated, i.e. $Ra \leqslant 1.5 \times 10^5$ and $0.71 \leqslant Pr \leqslant 130$.

1. Introduction

Rayleigh–Bénard convection in confined enclosures has been extensively studied (Yang 1988; Koschmieder 1993; Getling 1998; Bodenschatz, Pesch & Ahlers 2000) because of its practical applications and theoretical relevance. Since natural convection constitutes a fluid flow problem with well-established base solutions